# Natural treatment systems for waste water

## Basics of Groundwater flow

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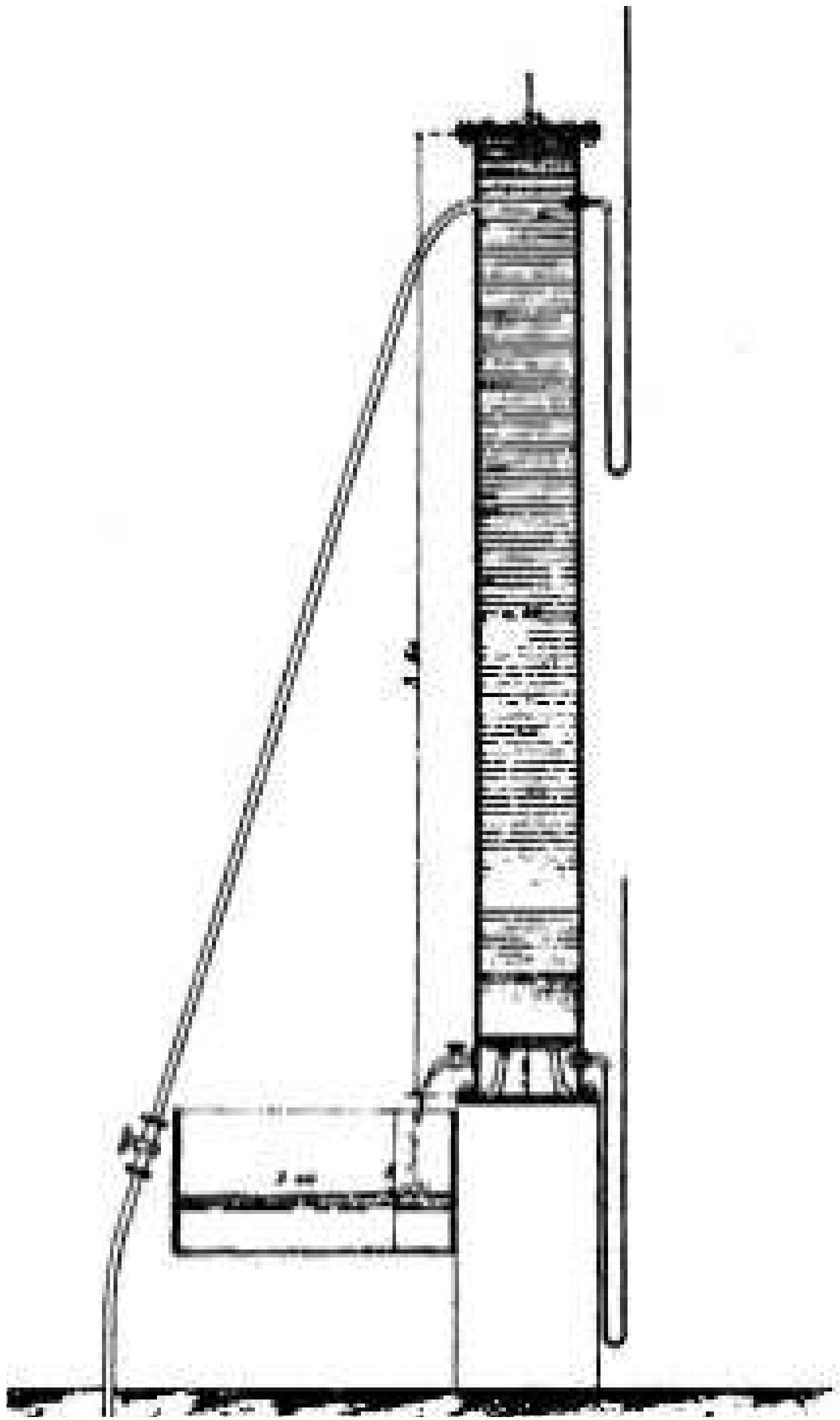
# Darcy's Law

- Henry Philibert Gaspard Darcy
- June 10, 1803 – January 3, 1858
- Scientist and engineer

Advies- en ingenieursbureau



# Darcy’s experiment

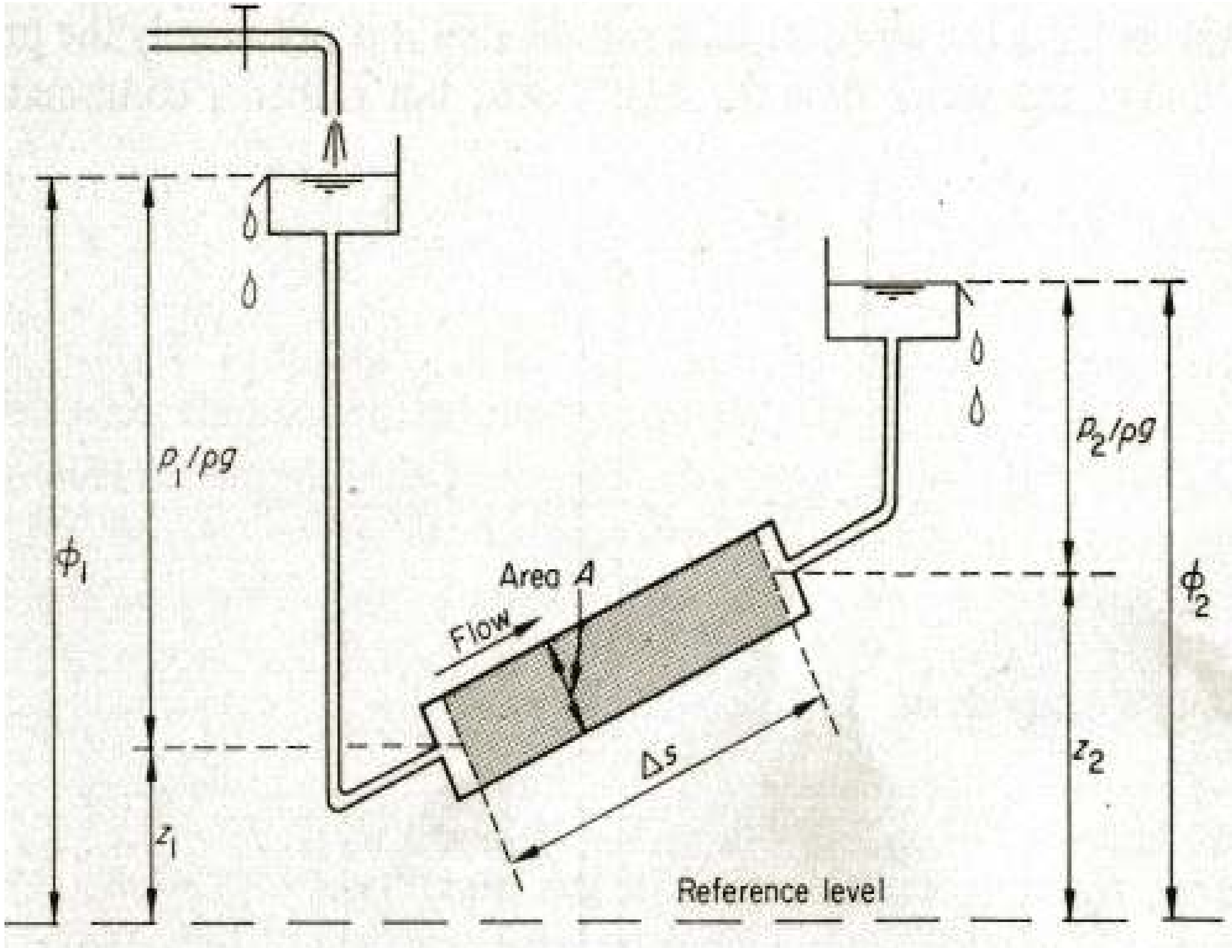
Carried out in 1855-1856

Column filled with sand and water

Measurements:

Flow rate [$L^3$ $T^{-1}$] (m3/sec)

Gradient in waterhead [L] (m)




# Darcy's Law

$$Q = k A \frac{\varphi 1 - \varphi 2}{\Delta s}$$

$$v = Q/n$$

$$T = \Delta s/v$$

Q: Flow rate [$L^3/T$]

v: Flow velocity [L/T]

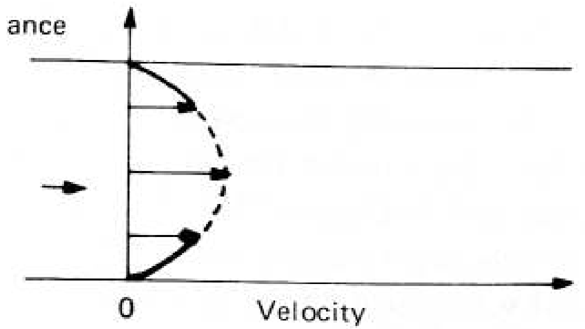
A: Cross section of column [$L^2$]

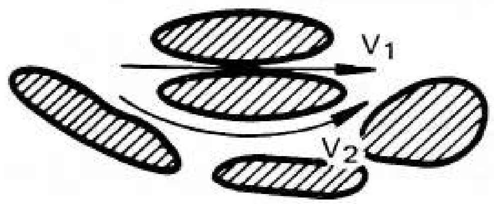
φ1, φ2: Head [L]

k: Hydraulic permeability of porous medium [L/T]

n: Effective porosity [-]

# Flow through porous media




Characteristic hydraulic permeability

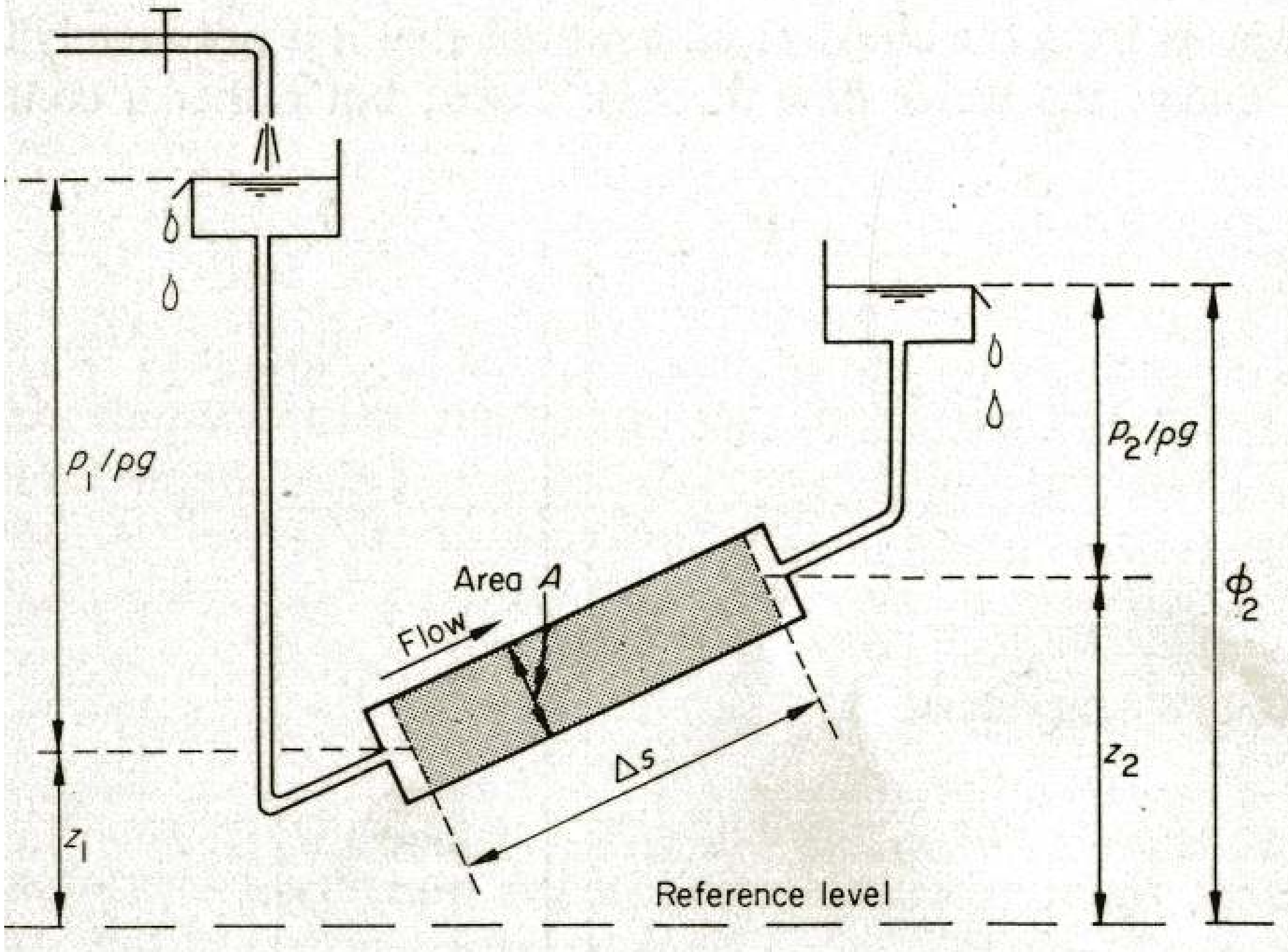
| | |
|---|---|
| Gravel | 100 - 500 m/day |
| Sands | 10 - 50 m/day |
| Loam | 0.1 – 1 m/day |
| Clay | 0.001 – 0.1 m/day |

Effective porosity 0.2 – 0.3

Natural flow velocities are limited (0.1 – 100 cm/day)

# Further reading?

- Theory of Groundwater Flow (A. Verruijt, 1970)
- Quantitative hydrogeology for Engineers (G. de Marsilly, 1966)
- Dynamics of Fluids in Porous Media (J. Bear, 1972)

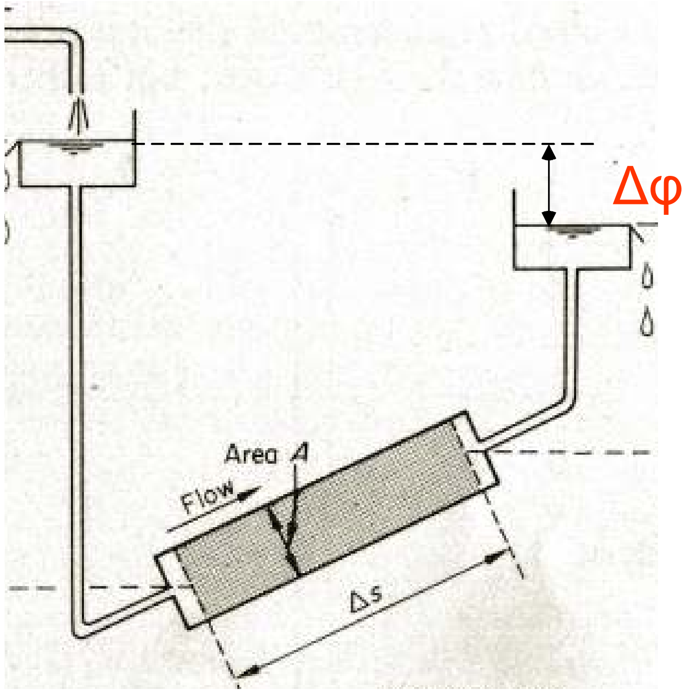



$$\varphi = z + \frac{p}{\rho g}$$

φ: Head [L]
z: Height [L]
p: Waterpressure $[M/L/T^2]$
ρ: Density of water $[M/L^3]$
g: Acceleration of gravity $[ML/T^2]$

# Darcy's Law






$$Q = k\,A\,\frac{\varphi 1 - \varphi 2}{\Delta s}$$

$$\varphi 2 - \varphi 1 = \Delta\varphi$$

$$v = Q/A$$

$$v = -k\,\frac{\Delta\varphi}{\Delta s}$$

v = specific discharge [L/T]

limiet Δs->0 $\quad v = -k\,\dfrac{d\varphi}{ds}$

In all directions

$$\begin{cases} v_x = -k\,\dfrac{\partial\varphi}{\partial x} \\ v_y = -k\,\dfrac{\partial\varphi}{\partial y} \\ v_z = -k\,\dfrac{\partial\varphi}{\partial z} \end{cases}$$

# Mass balance

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

If ρ=constant,

(dρ/dx=dρ/dx=dρ/dz=0), then:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Darcy:

$$v_x = -k\frac{\partial \varphi}{\partial x}$$

$$v_y = -k\frac{\partial \varphi}{\partial y}$$

$$v_z = -k\frac{\partial \varphi}{\partial z}$$

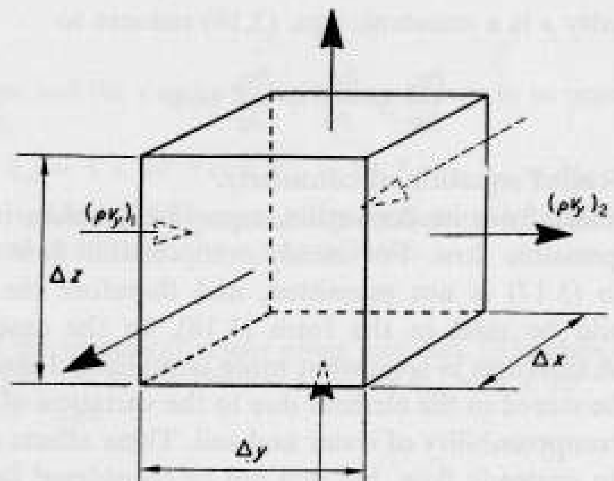



FIG. 3.4 Specific mass discharges into and out of element

$$\frac{\delta}{\delta x}\left(k_x\frac{\delta \varphi}{\delta x}\right) + \frac{\delta}{\delta y}\left(k_y\frac{\delta \varphi}{\delta y}\right) + \frac{\delta}{\delta z}\left(k_z\frac{\delta \varphi}{\delta z}\right) = 0$$

Diffusion equation for behavior of fluids in porous media

# Modflow

## CHAPTER 2

## DERIVATION OF THE FINITE-DIFFERENCE EQUATION

### Mathematical Model

The three-dimensional movement of ground water of constant density through porous earth material may be described by the partial-differential equation

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - W = S_s\frac{\partial h}{\partial t} \qquad (1)$$

where

$K_{xx}$, $K_{yy}$ and $K_{zz}$ are values of hydraulic conductivity along the x, y, and z coordinate axes, which are assumed to be parallel to the major axes of hydraulic conductivity ($Lt^{-1}$);

$h$ is the potentiometric head ($L$);

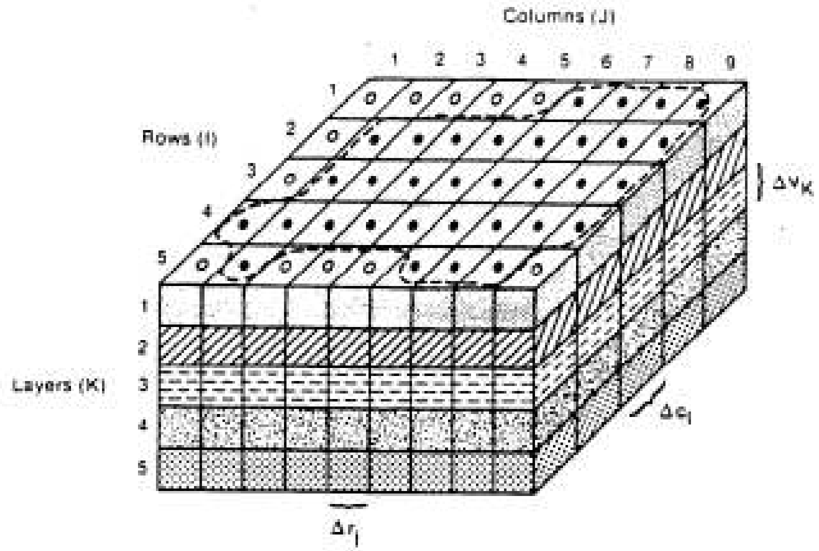
$W$ is a volumetric flux per unit volume and represents sources and/or sinks of water ($t^{-1}$);

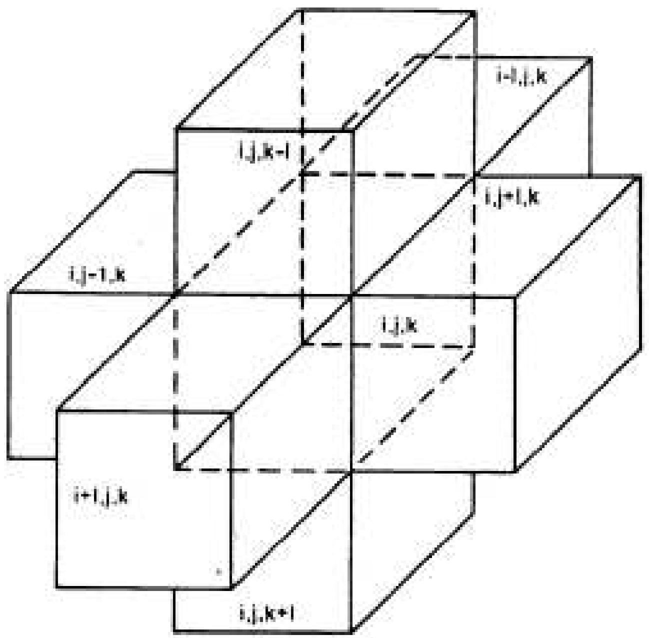
$S_s$ is the specific storage of the porous material ($L^{-1}$); and

$t$ is time ($t$).

For a derivation of equation (1) see for example Rushton and Redshaw (1979).

# Solution with finite differences




## Differential equation

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - W = S_s\frac{\partial h}{\partial t}$$

→

$$\begin{aligned} & CR_{i,j-1/2,k}(h_{i,j-1,k} - h_{i,j,k}) + CR_{i,j+1/2,k}(h_{i,j+1,k} - h_{i,j,k}) \\ & + CC_{i-1/2,j,k}(h_{i-1,j,k} - h_{i,j,k}) + CC_{i+1/2,j,k}(h_{i+1,j,k} - h_{i,j,k}) \\ & + CV_{i,j,k-1/2}(h_{i,j,k-1} - h_{i,j,k}) + CV_{i,j,k+1/2}(h_{i,j,k+1} - h_{i,j,k}) \\ & + P_{i,j,k}h_{i,j,k} + Q_{i,j,k} = SS_{i,j,k}(\Delta r_j \Delta c_i \Delta v_k)\Delta h_{i,j,k}/\Delta t. \end{aligned} \qquad (2$$

# Modflow

- Open source software
- Published by US Geological Survey (1988)
- Well documented and tested
- Worldwide many thousands of users (Research institutes, Universities, Engineering firms, Water companies)
- Further reading http://water.usgs.gov/software/ground_water.html
- For downloading instructions and source code